The Estimation of the Minimum Detectable Activity from Measured Passive Neutron Coincidence Counter Data

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Abstract

We describe the method applied within Canberra's waste assay software, NDA2000, for estimating the Minimum Detectable Activity (MDA) following a Passive Neutron Coincidence Counter Assay. In order to clarify the approach and aid the explanation we illustrate its use by giving numerical examples. The MDA algorithm shows explicitly how design and operational parameters can be used to influence the detection limit. This enables the key factors to be identified and balanced in a structured way, when establishing the compromises between often conflicting requirements at the start of instrument design or measurement programs that target a given performance. We extend the discussion beyond the usual confines of ideal statistical counting distributions. Finally, we briefly comment on the role of Advanced Waste Analysis (AWA) algorithms available within NDA2000 involving statistical filtering and multiplicity truncation in relation to stabilizing the MDA estimates and overall system performance in real-life measurement situations.

Introduction

Fundamental to the design and application of non destructive assay instruments for the assay of nuclear materials is the question of how small a quantity can be reliably detected (in this context activity or mass is implicitly used interchangeably - the two being simply related). The term "Minimum Detectable Activity", or simply MDA, is most commonly used to refer to this quantity. The term "Lower Limit of Detection" or LLD, or depending on context "Minimum Detectable Concentration", MDC may also be used. The MDA is often used as one of the many Figures of Merit (FoM) by which different systems may be compared. To be useful in this sense, however, it must be suitably defined in terms of probability theory and quoted under standard measurement conditions - both of which must be carefully stated. To ensure a fair comparison between counter designs, all of the underlying assumptions must be clearly stated. In this work we discuss passive detection limits in passive neutron coincidence counting. Extension to totals neutron counting is trivial. Active detection limits are more involved as has been discussed elsewhere [1-3] yet many of the points we wish to make are generic to all methods.

Method

Several formulae are in routine use for calculating MDA values [4] reflecting the fact that the selection of a particular approach depends on the application and on the perceived end use of the information. For example, the selection would depend on whether one is permitted to form a consignment average taken across a group of drums treated as one or if the single item assay is of paramount interest. Different options are frequently selected or prescribed by various researchers, standards committees and regulatory bodies depending on the particular assay situation. When using the MDA as one FoM by which to compare different instruments and measurement regimes it is important that a common approach is adopted.

To cite a concrete example, within Canberra's NDA2000 waste assay software, the **passive** neutron MDA, expressed in mg of $^{240}Pu_{eff}$, for Reals coincidence counting is calculated via the following expression:

$$m_{MDA} = \frac{2.71 + 4.65 * \sqrt{R_{eff\_bkg} \cdot t}}{a_R \cdot t} \quad (1)$$

where

a_R = Reals 'calibration' parameter or specific response, Reals.s^{-1}.mg $^{240}Pu_{eff}^{-1}$

t = assay count time, s, taken for the present discussion to be equal for both the background and item counts.

R_{eff_bkg} is the "equivalent" or "effective" Reals background rate.

The effective Reals background rate (expressed in Reals.s^{-1}) is obtained from the following expression:

$$R_{eff\_bkg} = T^2 \cdot G + R_{bkg} \quad (2)$$

where

T is the Totals neutron count rate, Totals.s^{-1}, during the item assay - not the background assay.

R_{bkg} is the measured net Reals background rate, Reals.s^{-1}, from the background assay.

G is the shift register gate width in units of s.

The T^2.G term is the expression for the Accidentals coincidence rate during the assay; it is included to account for the effect of potentially high Totals rates during the assay. High Totals rates might be encountered during the assay of a high (α, n) waste source or if the assay system is in a room of high ambient background such as a MOX or waste storage facility. Typically there is very little difference between the use of T (i.e. the Totals rate during the assay) and the use of T_{bkg} (i.e. the Totals rate during the background assay) in this expression. This is because in practice when counting at low levels (i.e. close to the detection limit) any contribution to the Totals count-rate from the Special Nuclear Material (SNM) present in the container is minor compared with the ambient rate. Thus, the difference between the use of T or T_{bkg} in the expression is often of second order importance. However the use of T above is explicitly a little conservative.

This expression estimates the so called MDA_{95} value meaning that the false positive (Type I error) and false negative (Type II error) probabilities have both been set to 5% (corresponding to a standard variate of a one-sided Gaussian distribution n=1.644853... from which the values of the other parameters follow: $n^2 = 2.706$, $2 \cdot \sqrt{2} \cdot n = 4.652$). The expression is a simplified formulation based on Currie's derivation [5]. It *assumes* that the background count time is long compared to the assay count time so that R_{bkg} can be taken as a known parameter. The expression is derived on the assumption that the counting statistics can be adequately represented by the Poisson distribution. This means that the mean, m, and variance, σ^2, in the distribution of counts based on an observation of N counts may approximated by m=N and σ^2=N respectively. Additionally, the number of counts, N, is assumed to be sufficiently large (for both the background count and item count) so that the tails of the distribution can further be approximated by the Normal (Gaussian) distribution for the purposes of establishing confidence limits in terms of multiples of the standard deviation (std. dev. = $\sigma = \sqrt{(\sigma^2)}$).

In many cases of practical interest in quantitative gamma and neutron radio-assay these conditions are met sufficiently well for first order work but it is good practice to check on a case by case basis. We note in passing that the Gaussian approximation to the Poisson distribution is generally poor for more than about 3σ from the mean regardless of the mean N. This should be borne in mind if small values for false positive and false negative probabilities are of interest. As stated the formula is specific to the case of a background count and assay count of equal duration. It can be shown from quite general arguments [4] that for a given amount of available time for an individual assay that a time spent counting the background should be equal to the time spent counting the item. This applies if the samples are suspected of being free from activity and the objective is to achieve the lowest MDA. (If the items are known to have a net count rate which exceeds the background rate however, this partition of time would not deliver the optimal precision - in such cases more emphasis should be given to counting the item).

There may be other valued and valid reasons for setting the assay count and background count durations equal (i.e. $t_s=t_b$). One could argue, for example, that it is good practice for the background and item count sequences to be as similar as possible in all respects. This applies to the complete measurement sequence, including the actuation of all moving parts and the path through the control software, including the control of the scaling devices and the seeding of the algorithms. As an aside, we note that in certain gamma-spectroscopy applications the background beneath a peak ROI may be estimated from the counts in neighbouring ROI. In this case the background and assay times are naturally equal. On other occasions where the gamma-line of interest may also be present in the background a separate count may be necessary. For a fuller account of the assumptions and mathematical derivation the interested reader is referred to the article by Lochamy which provides an excellent companion to the paper of Currie [6]. In Annex 1 we make some additional observations of both theoretical and practical importance.

We note for completeness that the term $\sqrt{(R_{eff\_bkg} \cdot t)}$ is an estimate of the standard deviation in the (effective) number of coincident Reals background counts. Where data has been acquired as a sequence of repeat count cycles this can be replaced by an estimate based on the standard error of the sequence data. In this way, allowance can be made for any additional variation that may be present due to the inherent 'lumpiness' (i.e. non-normal distribution) of the cosmic ray related component. Alternatively, an additional empirical multiplier to the $\sqrt{()}$ term could be factored in to allow for this in an empirical but generic way.

Example 1

Consider a hypothetical combined passive-active neutron waste assay chamber with a neutron detection efficiency of 26% and a Reals calibration parameter of 0.037 Reals.s^{-1}.mg^{-1}. Further suppose the chamber is operated under conditions and in a location where the ambient background rates are on average 20 Totals.s^{-1} and 3 Reals.s^{-1}. For a massive graphite wall chamber designed to handle heavy 400 litre drums this may correspond to a spacious room with minimal overhead shielding. Given this scenario what is the MDA (=LLD_{95}) for a 30 min total assay period?

From experience or pre-calculation, we estimate that the MDA will be in the mg range for $^{240}Pu_{eff}$. With a small quantity of non-multiplying (e.g. no large quantities U or Be, say, present and in close contact with the Pu) but impure reactor grade Pu (30wt% $^{240}Pu_{eff}$, α=(α,n)-to-(SF,n) production rate in the SNM=5) present, the net Totals rate from 5mg of $^{240}Pu_{eff}$ is expected to be about: 0.26*0.005*1023*(1+5) = 8cps (where 1023 is the number of SF neutrons emitted per sec by 1g of $^{240}Pu_{eff}$).

Therefore, as an initial estimate we use a value of T equal to 28 Totals.s^{-1}. For a coincidence gate width, G, of 128μs the Accidentals rate during the passive assay will be about ($(20+8)^2$ x 128x10^{-6} =) 0.10 Reals.s^{-1} i.e. only 3% of the ambient Reals background rate in this example which underscores the assertion in the main discussion that for some class of problems (although by no means all – depending on the counter, materials being assayed and environmental conditions!) the exact treatment of Accidentals is of secondary importance.

Suppose that out of the nominal 30min assay period time only 24min is available for actual counting given the overhead of drum handling, reading of the data, statistically filtering the data and associated loss of cycles, printing of the reports and so forth.

We are now in a position to use the given expression to estimate the MDA under the conditions laid out:

$$MDA = [2.706 + 4.652 \cdot \sqrt{((0.10 + 3.00) \cdot 1440)}] / (0.037 \cdot 1440) = 5.9 \text{ mg } {}^{240}Pu_{eff}$$

Simple Scaling Rules:

Note that in this example the leading term (2.706) makes only a small contribution which is characteristic in this count-rate regime. Therefore, to halve the MDA would take about four times as long - the familiar root-t scaling rule. Similarly, to halve the MDA by a factor of two would require the ambient background to be lowered by roughly a factor of 4. This would (typically) require substantial non-coupled overhead shielding. To achieve a performance improvement of this magnitude through initial design of an enhanced performance assay system would probably be challenging and would most likely require the careful review of all materials and their proximity to the sensitive ^{3}He proportional counters. The number, placement, construction and fill pressure of the counters as well as intrinsic shielding provided by the walls of the counter would also need to be carefully addressed. Considerations such as these are at the heart of bespoke instrument design targeting performance-based requirements.

The Significance of AWA:

NDA2000 supports several Advanced Waste Analysis (AWA) options, for example: AAS matrix correction and AAS moderator compensation of the background; multiplicity analysis including the three variants One's Rate, Truncated Doubles and Truncated Multiplicity; High-Z Matrix analysis. A full discussion of these techniques is outside the scope of the present work (see the NDA2000 Technical Reference Guide for further information). We confine our remarks around the general axioms surrounding the use of truncation close to the detection limits.

In practice the cosmic ray background fluctuations tend to be greater than root-N counting statistics would suggest. This is because the number of primary or initiating events is less than the total yield of neutrons they spawn. In particular the larger multiplicity events are more sporadic and so would be described by a broader distribution, one with a higher variance. For heavy drums of high atomic number materials this can be a significant problem, especially at altitude. By switching to One's counting at low Pu mass levels the detection limit is not reduced much. This is for two reasons. The first is related to the fact that some genuine Two's and higher events are also rejected so that the specific response is therefore lowered by perhaps 20%. The other factor however is that the MDA calculations do not properly account for the cosmic events. The approach of using the count-rates and not the observed standard deviations is to underestimate the uncertainty by 10%-20%. This leads to an optimistic MDA estimate. The truncation eliminates much of the non-Gaussian component making the count-rate based approach a more accurate estimate. But, what the use of One's counting does mean therefore is that the MDA-formula prediction is a more reasonable and a more robust estimate since the fluctuations are 'better behaved' - that is more in keeping with Poisson counting statistics based on the number of counts observed.

Another minor effect should also be considered. When data is rejected by the statistical filters acting on the repeat cycle structure the effective data acquisition time is also reduced. Thus although the data set becomes tighter the measurement time has effectively been reduced. The counting sequence and rejection criteria must therefore be chosen to produce a net benefit without undue distortion. Typically a count structure which results in a few percent of cycles rejected based on 2.4σ to 3σ rejected is considered suitable.

Example 2

As a rough estimate suppose going over to the use of One's reduces the specific response by 20%, that is to say:

$$a_1 = 0.8 \cdot a_R$$

and further suppose that as a consequence the coincidence background rate is reduced by a factor of 0.55 with little change in the Totals rate. The estimated MDA in this case would be reduced marginally to approximately 5.5mg $^{240}Pu_{eff}$ as calculated using the full MDA expression and other parameters as discussed above (we note that this is also in very good accord with the approximate scaling rules for this counting regime i.e. [$\sqrt{(0.55)/0.8}\cdot 5.9$]).

Note that achieving a much lower MDA, a 1mg detection limit say, in any reasonable time (e.g. a 4 hr assay supported by a 4 hr background measurement) using the system in the illustrative example is likely to be most challenging. It would probably require major revision of the assay system and the addition of substantial shielding.

Managing MDAs

Equation (1) illustrates that there are only three fundamental ways to govern the MDA, namely:

i. low activity measurements invariably require, in as far as is possible, a suppression of the counting rate from all but the non-targeted nuclide. This can be via environmental management, active or passive shielding combined with discrimination by such methods as statistical filtering of which high multiplicity rejection is one example.

ii. Count sequence - take advantage of all the time available, maximise the amount of time available for background and item counting and partition that fraction to best advantage between the two. Time spent handling items and performing non-counting tasks should be reduced as far as practical without compromising safety or quality control.

iii. The specific response of the instrument should be optimised for the specific items under consideration with economic and other constraints. For active interrogation methods, source brightness as well as detection efficiency is important. Where the interrogating source strength is variable (as a result of radioactive decay or operational decline, say) care should be taken limit the useful life of the source, alter the counting sequence and/or periodically trim the brightness during prolonged campaigns. When designing a new instrument from scratch there is clearly a strong interplay between sensitivity, geometry, background, throughput etc. and it is part of the design teams goal to achieve a near-optimal compromise of these factors when taken together with the other constraints that apply to the design process.

Evaluating Uncertainties on the MDA Estimate

Equation (1) is the basic relationship by which the MDA is estimated, it also provides the means by which the uncertainty in the MDA estimate can be estimated given the uncertainties in the contributory factors. In taking this step we explicitly recognise that the MDA is only an estimate based on estimates and furthermore depends entirely on the underlying assumptions associated with the measurement and analysis.

With obvious notation we can re-write Equation (1) as follows:

$$MDA = y = [2.71 + 4.65 \cdot \sqrt{(B.t)}\ ] / a.t$$

In general the timing uncertainty is negligible in relation to other sources of uncertainty. Furthermore we may assume that for a particular assay *B* and *a* are uncorrelated (clearly when estimating the performance of a new design this is not the case, but once a system is in operation it is).

Thus, the estimated standard deviation σ_y on y may be evaluated according to the familiar quadrature sum rule as:

$$\sigma_y = \sqrt{(((\partial y/\partial B).\ \sigma_B)^2 + ((\partial y/\partial a).\ \sigma_a)^2)}$$

where

$$\sigma_B = \sqrt{((2.T^2 G.(\sigma_T/T))^2 + \sigma_{Rbkg}{}^2)}$$

and σ_a is the estimated uncertainty, at the one standard deviation level, on the calibration constant for the (waste) item in question. This depends on the quality of the initial absolute calibration data including the knowledge of the effective mass(es) (or activities) of the reference materials used and the calibration procedures and processes adopted. In general for a given item the value of *a* also depends on the validity of various correction factors applied

a reference case calibration. These factors may be derived from experimental data and themselves be subject to statistical and calibration uncertainties or may be based on separate analysis or on additional assumptions. For instance the gross matrix correction may depend on a combination of Add-A-Source (AAS) and flux monitor information combined according to a previously defined scheme with an associated uncertainty budget. Often if lumps of fissile material are suspected, then a calculated allowance based on acceptable knowledge may be applied. There may also be additional uncertainties to do with how the data was statistically filtered or corrected for high atomic number or moderator perturbation of the background. These are best evaluated empirically by sampling.

Perhaps the most dramatic variation to consider however is the basis used to select the value of *a* itself. The value of *a* reflects the basic assumption of source distribution assumed (since at the detection limit for the system it is generally not possible to experimentally discriminate non-uniform source distributions based on partitioning of the observable counts between detector banks - or views). The wide spread and traditional assumption is to adopt the calibration factor appropriate to a uniform distribution. On the average, when taken across an ensemble of similar items each assumed to be randomly packed, this would yield an accurate answer. The conservative estimate for a particular item would take the lowest (worst case position) *a*-value estimated. The optimistic MDA would, conversely, be based on using the largest value of '*a*' which amounts to assuming that the activity is localised in the zone of the item which is most easily 'visible'. The traditional volume weighted average MDA estimate lies between the inherently safe or pessimistic value and the optimistic value. All three values are equally valid figures to report provided they are accompanied with the appropriate commentary. The 'best' selection depends on the end use to which the information will be put.

The important points we wish to emphasise here, are that the MDA commonly referred to in the Non-Destructive Assay (NDA) literature, is rooted in counting statistics and is but one FoM from many that could be used; that it is itself reliant on the assumptions made in its definition and application; and is, furthermore, subject to (sometimes large systematic) uncertainties. Although amenable to quantitative study, the currently applied state of the practice does not account for the complete level of complexity hinted at above. However the regulatory bodies responsible for oversight, authorising shipment of, or receiving waste items do not have a means of dealing with such refined analysis even if it could be provided on an item by item basis. Because of this it is our opinion that the MDA estimates should rightly be treated as indicative information to be used by suitably qualified and experienced subject matter experts in the proper context of the assay rather than as hard definitive values to be applied in a sterile theoretic context.

Future Work

A similar discussion is needed in relation to the reporting practices commonly used within the waste assay community. The use of Detection Limits and Less-Than values requires some clarification. We invite interested readers to contact the authors with their views on this very important topic.

Conclusions

An important a priori figure of merit for a non-destructive assay instrument is the Minimum Detectable Activity. The MDA can also be derived from measured data for every assay. A review of how the MDA is calculated within NDA2000 has been presented and examples in passive neutron coincidence counting have been provided to clarify how the method is applied in practice. We have discussed how the MDA should be treated and interpreted within the context of waste assay which is not a simple (gross) counting experiment and therefore differs from the accounts that may be found in statistical texts.

The factors influencing the MDA are discussed. The discussion includes an informative section on how to manage MDAs. The approach used to place an uncertainty on the MDA estimate is described including allowances for contributions not related to conventional counting statistics. This aspect is often ignored in most other treatments.

It is noted that the MDA is one of several figures of merit available to describe the effectiveness of an assay and is itself subject to uncertainty the appreciation of which may require review by a suitably qualified and experienced subject matter expert.

Annex 1: Additional Observations and discussion

In their discussion Hurtgen et al make the following statement about counting data:

"...the distribution of data will tend to a binomial rather than a Poisson distribution at low activity..."

and go on to make the point that as a consequence, at low counts, the standard deviation on N counts is $\sqrt{(N+1)}$ as obtained from the binomial distribution in contrast to $\sqrt{(N)}$ as obtained from the Poisson distribution. To support this statement the authors reference the text by Friedlander et al [7].

We consider this statement to be extraordinary. It is true that the Poisson distribution is a limiting case of binomial distribution although it can also be derived directly. The basic Poisson assumption is that there are a very very large number of trials and a very very small and constant probability of success associated with each. This is exactly what is assumed for counting of nuclear decays etc. We are not sure where "low counting statistics" changes any of the assumptions so as to invalidate the Poisson assumption. In other words we assert that, for all practical purposes, in nuclear counting experiments of interest to the radio-analyst the Poisson distribution should be an adequate representation of pure counting statistics. (The binomial distribution is of more general application than either Gaussian or Poisson and is suitable where the probability of success is *not* small, for example if the likelihood of counting a nuclear event, p, is sufficiently large that (1-p) cannot be approximated to 1. This would apply to recording (a small number of) nuclear events with high probability of detection.) Of course there are other processes that may come into play that are not governed by the Poisson law. Important examples drawn for both quantitative gamma-ray spectroscopy and neutron counting, include:

- cosmic ray variations during the counting period. There are well known diurnal cycles and variations associated with sun spot activity and solar storms. On the short term changing weather patterns can have a significant effect with the cosmic ray generated neutron background falling by about 0.8% - 1% per mmHg rise in atmospheric pressure.
- Rn emanations for the ground into the counting shield, these can follow atmospheric conditions
- RA decay of other short live nuclides in the sample, a common problem when counting activation products
- the deviation between the item and the ('representative') blank specimen
- the deviation between the item and the items used in the calibration
- the item may modify the background signature and the allowance for this may introduce additional uncertainty
- it is usually assumed that timing uncertainties are negligible in comparison to other sources of uncertainty, but, the value of the calibration parameter, *a*, may be subject to experimental uncertainties associated with gross characterisations of the item (e.g. the AAS matrix correction) and also on unknown properties of the item (i.e. it is traditional to assume a homogeneous matrix and a uniform activity distribution with the item - deviation from these assumption mean that the true value for the calibration parameter may deviate from that adopted). The correction factors for these effects introduce uncertainty including, maybe, a time dependence. For instance the decay correction of the AAS reference rate may introduce a small bias as the time between the reference date and the assay date extends.
- discriminator drift (e.g. threshold in the case of counting experiments or Region of Interest boundaries in the case of spectroscopy). This may be correlated to other factors, such as environmental temperature of the (electronics) device which may need to be controlled, or it may show aspects of independent variation.

- power supply stability, gain stability etc. which may depend (systematically or unpredictably) on ambient temperature and various other factors such as other electrical operations taking place in the facility. Without suitable precautions the relative humidity may also influence electrical breakdown in HV components such as cables, connectors and amplifiers especially where small charge pulses are being handled.
- intermittent low level electrical noise
etc...
but these influences are not the focus of the statement given by Hurtgen et al - since the discussion of MDA given there is kept purely on idealized statistical grounds. Of course a 'good' experimental design will seek to manage these other factors to either limit there impact or allow for them explicitly. One way of allowing for such factors in a general way is to partition the counting process into a sequence of short alternating contiguous background and item counts (respectively) and to use the scatter in the data itself to define the counting statistics (probability distributions) directly. Few situations call for this extreme approach and the assumption of a Poisson distribution is almost always made. None-the-less it is good practice to split each count into a sequence of shorter counts or repeat cycles and furthermore this can aid expert technical review. Furthermore it is good practice to confirm the stability of the system over durations typical of the period between the before and after backgrounds being used to sandwich the unknown counts. The fluctuations can then be factored into the analysis. Another precaution seldom used in practice is to also record the Temperature, Pressure and Relative Humidity and any other controlling influence throughout the campaign.

As for the specific claim in the paper of Hurtgen et al, that the standard deviation on a count of N should be $\sqrt{(N+1)}$ (rather than $\sqrt{(N)}$) as a direct *consequence* of binomial behaviour at low counts, we believe this to be simply a misunderstanding. Friedlander, the reference they cite in support of this statement, does not support it. The situation, is we believe, however, properly represented in Freidlander et al, [7].

In their discussion of the inverse problem of counting a Poisson distribution Friedlander et al credit RW Dodson although no reference is provided. The results presented are however in accord with those derived by Rainwater and Wu [8]. For completeness we summarise the main results here. Suppose we observe an actual count of N and we want to solve the inverse problem of that usually treated in text books when the Poisson distribution is being discussed. That is, we want to estimate the characteristic Poisson distribution parameter, *a*, from which the count came. The number of counts observed (recall this is not net counts) N is a positive integer. It may be a small number, even zero. The probability that the actual value of *a* lies in the interval d*a* about *a* is given by:

$$P_n(a).da = (a^N.e^{-a} / N!).da \qquad (3)$$

where *a* is a continuous, real, positive or zero variable and we are treating the increment d*a* as a differential.

The most probable value, a_o, of *a* is obtained by maximising Eqn(3), that is by setting the gradient of $P_n(a)$ with respect to *a* equal to zero and solving for *a*. This gives:

$$a_o = N \qquad (4)$$

This result states that the most probable value of the Poisson parameter ($a=a_o$) is the observed number of counts N.

The average or mean value of *a*, a_m, is evaluated according to:

$$a_m = \text{Integral}\{0, \text{Infinity}, a.P_n(a).da\} \qquad (5)$$

which gives:

$$a_m = (N+1) \qquad (6)$$

This result states that the mean (average or expected) value of the Poisson parameter (a=a_m) is larger than the observed count by one. Thus, if no counts are observed the expectation is that it would be finite and equal to unity if the experiment could be repeated many times.

It is straightforward to show that the expected variance of the distribution, σ^2, is given by:

$$\sigma^2 = a_m = (N+1) \qquad (7)$$

Based on these results, which in a Bayesian sense are derived under the assumption that all sample strengths are a priori equally probable, we conclude that given an observation of N counts the parent population may be described by a Poisson distribution with a characteristic parameter given by (N+1). In other words if we observe a count of N we would report our best estimate of the mean along with the standard deviation uncertainty band as:

$$a = "a_m \pm \sqrt{(\sigma^2)}" = (N+1) \pm \sqrt{(N+1)} \qquad (8)$$

Note, when differences are formed between counts of equal duration (item count less blank count) then the net count is unaffected by the addition of one to the expectation value.

If N>>1 the difference between N and (N+1) in these expressions is inconsequential for most practical purposes. When N is small this is not so. Therefore, strictly if we are talking about 'minimizing the variance' when optimising counting strategies then we believe Hurtgen et al should be using σ^2=(N+1) any way. That is to say it is **not** a direct consequence of the Poisson distribution being inappropriate at low counts so that the binomial distribution should be used. Rather it is a direct consequence of using Poisson statistics, which we re-assert, is a wholly appropriate distribution for most pure nuclear counting experiments [9].

We pose here a variant of the problem that was solved in Section 2 of Hurtgen et al. What Hurtgen et al do is give a *mathematical solution* to what is also a complex operational and Quality Control (QC) procedural problem. The mathematical solution does not address whether is it better to perform a background count of 1/4 the duration both before and after the item count rather than a single count of 1/2 the time available. In a series of counts we could extend this approach so that the background count at the end of one count becomes the start background count at the start of the next. We could also factor in the sample change over time and treat that as an additional control parameter (t_L or parasitic lost time).

But what is often done in practice, at least in waste assay, is that a background is taken at the start of the shift and another taken at the end of the shift. Sandwiched in-between are a number (7 say) of item counts. So the shift time should now be partitioned between 2 background counts and 7 (in this example) items counts. What is the best use of the available time now assuming the individual item assays are of principal interest (rather than the aggregate taken over all items of a consignment). In other words, practical considerations, local Quality Assurance (QA), QC and consequence analysis needs, and other often mundane factors also exert a great deal of influence on the way assay programs are constructed and measurement regimes conducted. For instance, if the 'back-end' background count is not consistent with the 'front-end' background count there is the problem of what to do. It may mean there has to be an output buffer store to hold the items before they can be released so that the consequences of any potential mis-assignments are contained. If such a store is impractical or the consequences so grave or the handling of the items so delicate a matter, then each assay may need to be self contained with its own before and after checks.

One may also ask why is it that 5% Type I and Type II uncertainty probabilities are selected in the first place. What is so special about this value? The answer is nothing! It is purely an historical and psychologically pleasing choice. By analogy we may arrange to meet friends for dinner at 7:30 but in practice 7:27 is every bit as good. But 7:27 is not a conventional choice. Similarly, setting α=β=5% is essentially an arbitrary choice. The user should have the freedom to set α and β independently (this in turn sets the corresponding values of n and the coefficients in the general formula used in estimating MDA. For example, α = 50% and β = 2.7% corresponds to a net signal equal to three times the (overall) statistical standard deviation (3-σ) which was a popular alternative definition in neutron

counting for a long time. In other areas of Physics, such as the discovery of a new fundamental particle, the burden of statistical proof is often required to be far more demanding than MDA_{95}. A single experiment claiming at effect at such a level would be treated an indicative and encouraging but little more. Independently repeated searches, preferably by means that don't share common sources of potential systematic bias or error would be sort. Improved experiments with better precision would be conducted. Only when corroborated results each with effects plain at lets say 6-σ would the scientific community en masse endorse the finding. Furthermore, the treatment of MDAs along with the protocol for reporting quantities below the MDA are often driven largely by nonsensical regulations that are not based on sound statistical logic. For example, when conventional (rather than Bayesian) statistical inferences are being made not carrying negative assay results can significantly bias a waste stream average.

It should also be noted that the MDA is the estimated answer to a specific question, namely, is activity present or not according to a particular statistical definition. It does not address the more general and perhaps more pertinent question of how accurately and with what Total Measurement Uncertainty (TMU) a given quantity of nuclide can be measured under the corresponding assay conditions. In this respect it is only one of several figures of merit of interest. It should not therefore carry more significance than it deserves. If you ask is my drinking water 'safe' and an authoritative assayist tells you that it is not radioactive at the MDA of a given instrument under a given set of measurement conditions, then that may satisfy a regulatory prescription but what does it *really* mean to you. It is natural for the lay drinker to ask for an improved measurement, one with 10 times better statistics say. Also to ask for better epidemiological data so that the risk can be placed into a better defined quantitative context. Further to request a better basic understanding of the physical health damaging mechanism. In other words, an MDA is a crude statistical quantity, which as often applied ignores the uncertainties in all of the item related corrections, it is also far from a complete description of complex decision making processes which are often itself poorly understood.

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