ICEM'05:
The 10th International Conference on Environmental Remediation and Radioactive Waste Management
September 4-8, 2005, Scottish Exhibition & Conference Centre, Glasgow, Scotland

ICEM05-1228

A SEMI-EMPIRICAL FORMULA FOR REPRESENTING THE FULL ENERGY PEAK EFFICIENCIES OF PLANAR GE DETECTORS

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ABSTRACT

The non destructive assay of special nuclear materials in storage containers or as waste items often calls for the full energy peak efficiency to be estimated in geometries which may be adequately described by the far field approximation. The principle energy range of interest for materials such as Pu + ^{241}Am and HEU is below 500 keV. Small volume LEGe detectors were once required for such applications where high resolution was needed to extract information from the complex spectral regions. Recently, large volume planar detectors called Broad Energy Ge (BEGe) detectors have become available. BEGe detectors offer high resolution at low energy, as with LEGe detectors, but with higher efficiency based on greater solid angle and thickness. However, significantly changing detector dimensions requires an examination of the adequacy of functions used to estimate and trend efficiencies. In this work we discuss how the full energy peak efficiency of such planar detectors may be estimated simply from the crystal and encapsulation dimensions. A new semi-empirical formula is presented for the calculation and parameterization of the characteristic far field full energy peak efficiency for energies below the pair production threshold. The formula is an extension of the any interaction model with allowance for the escape of fluorescent x-rays and secondary scattered radiation. A database of sixteen such planar detectors with a wide variety of dimensions, covering volumes from ~8 to ~150 cm^3, is used to validate and verify the method. In principal, this simple and quick to apply method is absolute given accurate dimensions of the active volume, thickness of any frontal dead layer and details of the end cap. Alternatively it can be used as a fitting function to interpolate or extrapolate experimental data. The results show that under the conditions examined, the formula is capable of reproducing measured efficiencies to within a few percent over the energy range of interest.

INTRODUCTION

The non destructive assay of SNM in storage containers or as waste items often calls for the full energy peak (FEP) efficiency to be estimated in geometries which may be adequately described by the far field approximation (source to detector separation much larger than detector dimensions) and sources primarily in front of the detector. The principle energy range of interest for materials such as Pu + ^{241}Am and highly enriched uranium (HEU) is below 500 keV. This range also covers the primary range for low energy mode relative isotopic measurements of Pu by multi-group analysis methods. Small volume LEGe detectors were once popular for such applications where high resolution was needed to extract information from the complex spectral regions. Recently large volume planar detectors, so called Broad Energy Ge or BEGe detectors, have become available. These offer high energy resolution at low energies as before but with higher efficiency because of solid angle and thickness.

In this work it is discussed how the FEP efficiency, $_{FEP}$, of such planar detectors may be estimated simply from the crystal dimensions and those of its encapsulation. The approach may be used to estimate the absolute efficiency from fabrication information, as a means to fit absolute experimental data or to fit relative efficiency profiles as would be called for in relative isotopics analysis codes.

BACKGROUND

A semi-empirical approach to accurately determining $_{FEP}$ aims to represent some key energy dependencies by functions with a physical foundation. In this way the general behavior can be captured using basic interaction data and knowledge of the crystal dimensions. A benefit of this approach is that fewer

free parameters are required and interpolation/extrapolation can be done with more confidence.

Consider a modified simple Any Interaction Model (AIM) approach [1]. The idea behind the AIM is that the total probability of interacting in the active detector volume can be evaluated using photon interaction coefficients and geometry information. The remaining task is to figure out what fraction of those events fall into the FEP. Intuitively we might expect the intrinsic FEP efficiency profile for normal incidence on a planar detector to be well represented by a function of the form:

$$\xi(\Omega, E, t_{att}, d, t) = f_{geom}(\Omega, E, t_{att}) \cdot \left(1 - e^{-\mu_{Ge} \cdot \rho_{Ge} \cdot t}\right) \cdot P(d, t, E) \quad (1)$$

where $f_{geom}(\Omega, E, t_{att})$ is a function to account for losses external to the active crystal region (solid angle and attenuation in absorbers including endcap and crystal dead layer), μ_{Ge} is the crystal mass attenuation coefficient, ρ_{Ge} is the crystal density, d is the crystal diameter, and t is the crystal thickness. In practice f_{geom} may be accommodated empirically by an exponential treatment of attenuators, perhaps with a parameterization of the mass attenuation coefficient energy dependence, and a numerical estimation of solid angle. The $(1-e^{-\mu \cdot \rho \cdot t})$ factor represents the total intrinsic efficiency. $P(d, t, E)$ is the intrinsic peak-to-total ratio. For an optically thick, large area (or collimated), detector one may expect $P(d, t, E)$ to be insensitive to dimensions (and consequently the volume) of the detector. In other words, we might be able to treat it as a universal function for a family of detectors which does not depend on the diameter, d, and thickness, t. This is like saying that at low energies and for a collimated beam the back face of the detector plays no role. A better approximation would include the dimensions of the detector. The peak-to-total ratio has strong energy dependence and it is therefore vital to represent this behavior especially above a few hundred keV. The challenge addressed in this paper is how to do this in a simple yet accurate and practically useful way.

Mowatt [1] writes that $(1-P_c)$ might be estimated by $\sim Q \cdot e^{-R \cdot E}$ where Q and R need to be determined for the detector at hand by fitting to experimental data. Here we prefer to keep the form more generally rooted to a plausible physical argument so that it is in principal absolute. We borrow from Cesana and Terrani [2] who indicate that $(1-P_c) \sim (1 - e^{-L \cdot b})$ where L is the crystal mean chord (taken as representative of the characteristic dimension) and b is an effective linear energy transport coefficient for Compton scattered secondary radiation in Ge evaluated at the incident energy. Some values of b(E)/4 are listed in reference [2] over the range 300 keV to 10 MeV. The results are suggestive that b(E) can be represented by a power law:

$$b(E) \approx a \cdot E^{b} + correction\ \ terms \quad (2)$$

or

$$ln(b) = A_{PT} + B_{PT} \cdot ln(E) + C_{PT} \cdot ln(E)^2 \quad (3)$$

thus, using the latter form we arrive at,

$$(1 - P_c) \approx 1 - e^{L \cdot b} = 1 - e^{L \cdot \left(A_{PT} + B_{PT} \cdot ln(E) + C_{PT} \cdot ln(E)^2\right)} \quad (4)$$

where A_{PT}, B_{PT}, and C_{PT} are characteristic coefficients to be determined once and for all and L is a geometry parameter specific to the detector.

Here we consider a refinement of the AIM – what we refer to as the Istvan-Wally Form (IWF). The original motivation was to obtain an approximate efficiency curve below 300keV for comparison with the functional form used in the MGA code [3, 4], but it extends up to the pair production threshold. If P_c is expressed in terms of Ge properties and dimensions of the detector it provides a generic, absolute, form. However, it may also provide a convenient empirical fitting function, within for example Canberra's Genie environment, by fitting for the free parameters entering into the expression.

METHODOLOGY

The full energy peak efficiency can be understood as consisting of two main components as shown in equation 5:

$$\xi_{FEP} = \xi_{geom} \cdot \xi_{intrinsic} \quad (5)$$

The attenuation component can be estimated using the standard exponential form that includes the material density, thickness/path length and energy dependent mass attenuation coefficient. The geometric component is also a measure of the solid angle as viewed from the emitting source. Point kernel and Monte Carlo methods can be employed to accurately estimate the attenuation and geometric components. For this study, with the assumption of far field geometry (our data is for source distance > ~ 300mm), the attenuation through the source encapsulation and endcap window were based on the measured thicknesses. We use the mass attenuation coefficient of Ge without coherent scattering. Coherent scattering is excluded since it does not result in energy deposition. The mass attenuation coefficient is a physical parameter and we can parameterize it any way we wish to achieve essentially arbitrary accuracy consistent with data evaluations. The solid angle efficiency component was determined via equation 6.

$$\varepsilon_{geom} = e^{\sum_i \mu_i(E) \rho_i t_i} \cdot 0.5 \cdot \left[1 - \frac{distance}{\sqrt{distance^2 + radius^2}}\right] \quad (6)$$

However, the component of primary interest to this study has been the intrinsic efficiency component. The intrinsic efficiency is the probability that all of the energy will be deposited once a full energy photon has entered the active region of the detector crystal. In the energy region below the threshold for pair production, the mechanisms for full energy deposition include either photoelectric absorption and/or Compton scattering. Photoelectric absorption can still not attribute to the full energy peak when subject to fluorescent x-ray escape, primarily impacting lower energies. The energy

dependent effect of such x-ray escape has been previously studied [5, 6] such that the photoelectric effect can be modeled per equation 7:

$$\xi_{PE} = \left(\frac{\mu_{PE}}{\mu_{PE}+\mu_{CS}}\right)\cdot\left(1-e^{-(\mu_{PE}+\mu_{CS})\rho_{Ge}\cdot length}\right)\cdot \left(1-e^{A_x+B_x\ln(E)+C_x\ln(E)^2+D_x\ln(E)^3}\right) \quad (7)$$

where μ_{PE} and μ_{CS} are the energy dependent Ge mass attenuation coefficients for photoelectric absorption and Compton scattering, respectively. The density of germanium, ρ_{Ge}, is 5.323 g·ml^{-1} and the *length* is the average active length of the crystal seen from the source. The x-ray escape coefficients have been set, based on our analysis, to the values: A_x = -13.2465, B_x = 11.3379, C_x = -3.4136, and D_x = 0.27799. The effect of fluorescent x-ray escape above about 130 keV is less than about 0.1%.

The probability that an incident full energy photon that does not initially interact by photoelectric absorption but still deposits full energy after Compton scattering must also be estimated. This effect is parameterized using a Compton peak-to-total function. This parameterization is made a function of both the incident energy and the average chord length of the active detector volume. As a first order approximation, this form assumes that the point of interaction might be anywhere in the active volume and the direction of scatter is more or less isotropic. The form is shown in equation 8:

$$\xi_{CS} = \left(\frac{\mu_{CS}}{\mu_{PE}+\mu_{CS}}\right)\cdot\left(1-e^{-(\mu_{PE}+\mu_{CS})\rho_{Ge}\cdot length}\right)\cdot \left(1-e^{-L\cdot e^{A_{PT}+B_{PT}\ln(E)+C_{PT}\ln(E)^2}}\right) \quad (8)$$

where L is the average chord length based on the active volume radius and length per equation 9:

$$L = 4\cdot\frac{volume}{area} = \frac{2}{\frac{1}{radius}+\frac{1}{length}} \quad (9)$$

and A_{PT}, B_{PT}, and C_{PT} are the peak-to-total coefficients, to be determined. The mean chord length is simply being used here as a convenient way to capture a characteristic dimension of the crystal. One can think of this dimension being used to index the macroscopic cross-section, b, for the scattered spectrum defining the escape probability. In summary, it is proposed that $_{FEP}$ can be described per equation 10:

$$\xi_{FEP} = \left(e^{\sum_i \mu_i(E)\rho_i\cdot t_i}\cdot 0.5\cdot\left[1-\frac{distance}{\sqrt{distance^2+radius^2}}\right]\right)\cdot \left(1-e^{-(\mu_{PE}+\mu_{CS})\rho_{Ge}\cdot length}\right)\cdot \left[\left(\frac{\mu_{PE}}{\mu_{PE}+\mu_{CS}}\right)\cdot\left(1-e^{A_x+B_x\ln(E)+C_x\ln(E)^2+D_x\ln(E)^3}\right)+\left(\frac{\mu_{CS}}{\mu_{PE}+\mu_{CS}}\right)\cdot\left(1-e^{-L\cdot e^{A_{PT}+B_{PT}\ln(E)+C_{PT}\ln(E)^2}}\right)\right] \quad (10)$$

The adequacy of equation 10 was experimentally verified. Measurements were taken with 16 detectors, selected to represent the full range of expected typical HPGe planar detector sizes (BEGe and LEGe). The selected detector type and dimensions are listed in Table 1. All of the measurements were taken with a point source in front of the detector face, centerline. The distance between the crystal face and source was well known through the use of a fixed jig. Attenuation from the acrylic source matrix and endcap window was based on nominal manufacturer thickness values and standard densities. All of the characterized planar Ge detectors have very little appreciable dead layer, expected to be in the range of 15-30 μm, and was not accounted for explicitly. However, the impact of this very thin attenuating dead layer affects primarily only low energies, is within the magnitude of measurement error and is accounted for implicitly by coefficient optimization. Mass attenuation coefficients were determined using the National Institute of Standards and Technology (NIST) XCOM utility at the particular energies, but could also be separately parameterized as functions of energy.

Table 1 – List of Selected HPGE Planar Detectors

Model	Window Material	Radius (mm)	Length (mm)
BE2020	carbon	26.76	18.68
BE2820	carbon	30.61	18.87
BE2825	carbon	30.20	23.64
BE3820	carbon	34.40	20.99
BE3825	carbon	35.67	24.25
BE3830	carbon	34.65	30.29
BE5020	carbon	38.37	27.16
BE5025	carbon	40.70	24.51
BE5030	carbon	39.14	32.04
GL0515	carbon	13.59	13.74
GL1015	carbon	17.75	14.50
GL2015	carbon	25.00	1628
GL2020	beryllium	24.75	21.00
GL2820	carbon	29.81	25.00
GL2825	aluminum	29.50	25.47
GL2830	aluminum	30.10	30.41

RESULTS

Figure 1 shows an example of a typical fit of the empirical function to measured data. All of the geometry dimensions and

attenuation parameters were measured and maintained as constant inputs. The three peak-to-total coefficients were then optimized by fitting to the measured data points (fits to a cubic gave no better fit). This was done with each of the detectors.

Figure 2 summarizes the results of optimizing each detector to respective measured efficiency data points using NIST traceable sources (^{241}Am + ^{152}Eu). The average efficiency ratio for all of the data points was 0.997 ± 0.019. For a given detector, the errors in the efficiency ratios using optimized coefficients did not seem to trend or be systematically biased with energy. The values for the three peak-to-total coefficients were tallied and the averages were determined. The average offset value (A_{PT}) was 8.381, the average slope (B_{PT}) was -2.433 and the average quadratic term (C_{PT}) was 0.119. These average values were then used to calculate efficiencies, with the resulting absolute efficiency ratios shown in Figure 3. The average efficiency ratio for all of the data points was 0.999 ± 0.062. For a given detector, the errors in the efficiency ratios using average coefficients did tend to be biased either high or low for all energies.

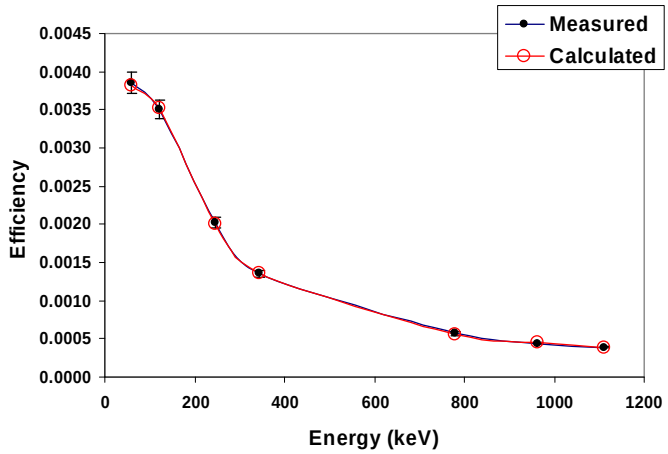

Figure 1 – Example calculated fit to measured efficiency data with optimized peak-to-total coefficients. Lines included to guide the eye. 1 standard deviation absolute experimental uncertainty bands are indicated.

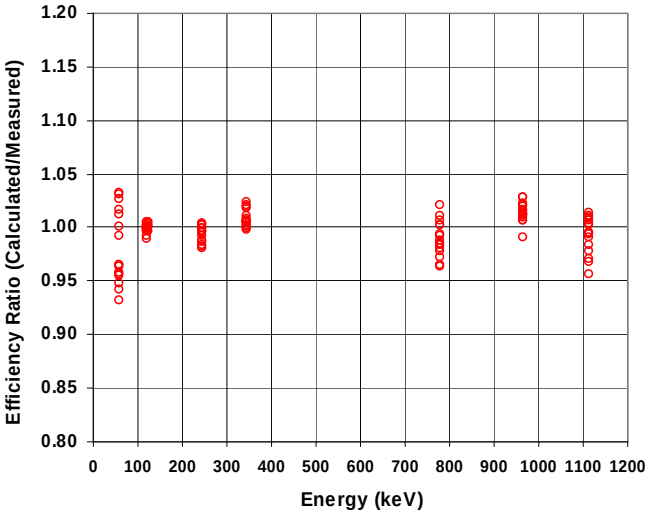

Figure 2 – Efficiency ratio results with optimized peak-to-total coefficients. The average efficiency ratio is 0.997 ± 0.019.

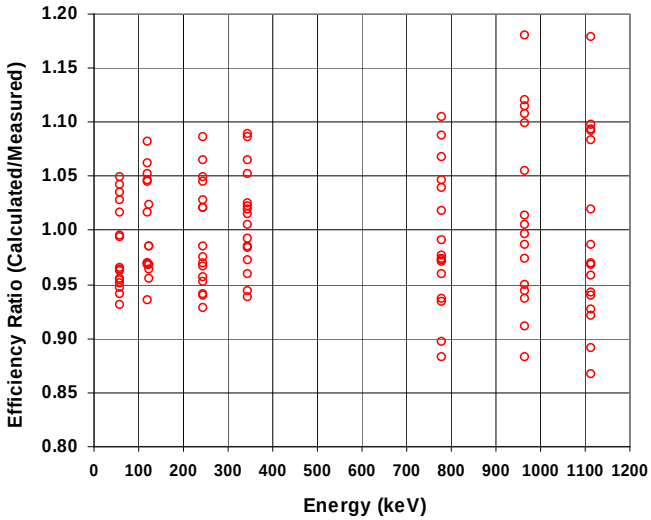

Figure 3 – Efficiency ratio results with average peak-to-total coefficients. The average efficiency ratio is 0.999 ± 0.062.

CONCLUSIONS

We have presented the development of the Istvan-Wally Form, IWF. Given accurate measured efficiency calibration data and attenuation and geometry parameters, efficiency calculations with accuracies within a few percent can be expected for planar HPGe detectors after fitting the peak-to-total coefficients, as was achieved in Figure 2. However, even if no efficiency calibration measurements are available, results within about 10% should be expected given accurate attenuation and geometry parameters while applying the average peak-to-total coefficients as per Figure 3. The functional form captures the underlying behavior of the primary interactions below the threshold for pair production and consequently also provides a means to fit the relative efficiency curve accurately with only a few free parameters. This could be of value in relative isotopic codes such as MGA. Future work will include exploring generalizing the method to the full range of detector types and a higher energy range by accounting for pair production. The use of the chord length introduces an approximation (weak attenuation) that might also be refined. For example, the detector radius and thickness or aspect ratio might be included explicitly to partially account for angular dependency effects of the secondary radiation.

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